Insight — Application Note 2.01

Terms and Definitions

$e = 2.71828$	Base of natural logarithm
$i = \sqrt{-1}$	Square root of -1
$\pi = 3.14159$	Pi
$\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm	Permittivity of free space
$k = 8.61733 \times 10^{-5}$ eV/K	Boltzmann's constant
f	Frequency (cycles/s or Hz)
$\omega = 2\pi f$	Angular frequency (radians/s)
q	Charge (coulomb)
t	Time (s)
σ	Conductivity ($\text{ohm}^{-1} - \text{cm}^{-1}$)
$\rho = 1/\sigma$	Resistivity, or ion viscosity (ohm-cm)
ε	Permittivity (F/cm)
$\varepsilon^* = \varepsilon_0 (\varepsilon' - i\varepsilon'')$	Permittivity as a complex quantity (F/cm)
$\kappa = \varepsilon' - i\varepsilon''$	Relative permittivity** or dielectric constant (unitless)
$\varepsilon' = \mathrm{Re}(\varepsilon^*/\varepsilon_0)$	Relative permittivity** or dielectric constant (unitless)
$\varepsilon'' = \mathrm{Im}(\varepsilon^*/\varepsilon_0) = \sigma/\omega\varepsilon_0$	Loss factor or loss index (unitless)
$D = \tan\delta = \varepsilon''/\varepsilon'$	Dissipation factor or loss tangent (unitless)
$\delta = \tan^{-1}(\varepsilon''/\varepsilon')$	Delta (unitless)
$\delta \sim \varepsilon''/\varepsilon'$	Delta for small $\varepsilon''/\varepsilon'$ (unitless)
A/D	Ratio of electrode area to distance between parallel plate electrodes (cm)
V	Voltage (volt)
$I = q/t$	Current (amp)
$C = q/V$	Capacitance (farad)
$G = V/I$	Conductance (ohm^{-1})
$R = 1/G$	Resistance (ohm)

** There is inconsistency in use of the term *relative permittivity*. In some literature it is the complex quantity $\varepsilon/\varepsilon_0 = (\varepsilon' - i\varepsilon'')/\varepsilon_0$ and in others it is the real part of permittivity $\mathrm{Re}(\varepsilon/\varepsilon_0) = \varepsilon'$.

$C = \varepsilon_0\, \varepsilon' A/D$	Capacitance (farad)
$G = \sigma A/D$	Conductance (siemen or ohm^{-1})
$R = \rho\, /(A/D)$	Resistance (ohm)
$Y = G + \mathrm{i}\omega C$	Admittance (siemen or ohm^{-1})
$Z = 1/Y = 1/(G + \mathrm{i}\omega C)$	Impedence (ohm)

α	Degree of cure (unitless, between 0 and 1)
α_{max}	Maximum degree of cure at cure temperature

M_{w}	Molecular weight (gm/mole)
M_{w0}	Molecular weight at $\alpha = 0$ (uncured)
$M_{\mathrm{w}\infty}$	Molecular weight at $\alpha = 1$ (fully cured)

T_{Cure}	Cure or process temperature (K or °C)
T_{g}	Glass transition temperature (K or °C)

ε' (free space)	= 1.0	Relative permittivity of free space
ε' (air)	= 1.0	Relative permittivity of air
ε' (teflon)	~ 2.2	Relative permittivity of teflon
ε' (mineral oil)	~ 2.2	Relative permittivity of mineral oil
ε' (polimide)	~ 3.6	Relative permittivity of polyimide
ε' (aluminia)	~ 9.8	Relative permittivity of alumina

DiBenedetto T_{g} model:

$$\frac{(T_{\mathrm{g}} - T_{\mathrm{g0}})}{(T_{\mathrm{g}\infty} - T_{\mathrm{g0}})} = \frac{\lambda\, \alpha}{(1 - (1 - \lambda)\, \alpha)}$$

Where:
T_{g} = Glass transition temperature (K or °C)
T_{g0} = Glass transition temperature at $\alpha = 0$ (uncured)
T_{gmax} = Maximum glass transition at T_{Cure}
$T_{\mathrm{g}\infty}$ = Glass transition temperature at $\alpha = 1$ (fully cured)
λ = Adjustable parameter

Debye relaxation for relative permittivity and loss factor:

$$\varepsilon' = \varepsilon'_{\mathrm{u}} + \frac{\varepsilon'_{\mathrm{r}} - \varepsilon'_{\mathrm{u}}}{1 + (\omega\tau)^2}$$

$$\varepsilon'' = \sigma/\omega\varepsilon_0 + (\varepsilon'_{\mathrm{r}} - \varepsilon'_{\mathrm{u}})\frac{\omega\tau}{1 + (\omega\tau)^2}$$

Where: $\varepsilon'_{\mathrm{u}}$ = Unrelaxed (high frequency) relative permittivity
$\varepsilon'_{\mathrm{r}}$ = Relaxed (low frequency) relative permittivity
$= 2\pi f$ (radians/s)
τ = Dipole relaxation time (s)

Havriliak-Negami relaxation for relative permittivity and loss factor:

$$\varepsilon' = \varepsilon'_{\mathrm{u}} + (\varepsilon'_{\mathrm{r}} - \varepsilon'_{\mathrm{u}})\,(1 + 2(\omega\tau)^{\alpha}\cos(\pi\alpha/2) + (\omega\tau)^{2\alpha})^{-\beta/2}\cos(\beta\theta)$$

$$\varepsilon'' = (\varepsilon'_{\mathrm{r}} - \varepsilon'_{\mathrm{u}})\,(1 + 2(\omega\tau)^{\alpha}\cos(\pi\alpha/2) + (\omega\tau)^{2\alpha})^{-\beta/2}\sin(\beta\theta)$$

Where: $\theta = \tan^{-1}[(\omega\tau)^{\alpha}\sin(\pi\alpha/2) / (1 + (\omega\tau)^{\alpha}\cos(\pi\alpha/2))]$
α = Empirical “broadness” parameter
β = Empirical “asymmetry” parameter
$\varepsilon'_{\mathrm{u}}$ = Unrelaxed (high frequency) relative permittivity
$\varepsilon'_{\mathrm{r}}$ = Relaxed (low frequency) relative permittivity
$= 2\pi f$ (radians/s)
τ = Dipole relaxation time (s)

Apparent relative permittivity and loss factor due to electrode polarization:

$$\varepsilon'_x = \varepsilon' \,(D / 2t_b)\, \frac{(\tan \delta)^2 + (D / 2t_b)}{(\tan \delta)^2 + (D / 2t_b)^2)}$$

Apparent relative permittivity

$$\varepsilon''_x = \varepsilon'' \,(D / 2t_b)\, \frac{(D / 2t_b) - 1}{(\tan \delta)^2 + (D / 2t_b)^2)}$$

Apparent loss factor

Where:
t_b = boundary layer thickness
D = distance between electrodes or plate separation
ε'_x = uncorrected permittivity
ε''_x = uncorrected loss factor
ε' = actual permittivity
ε'' = actual loss factor

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